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Square-Root Klein–Gordon Operator and Physical Interpretation

Kh. Namsrai[1]

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The square-root operator approach to relativistic quantum field theory is proposed. It is shown that an exact solution of this operator equation is a spinor with random mass distribution. A physical (potential) origin and gauge-invariant electromagnetic interaction of this new kind of particle are studied. Spreadout particles over mass value and space-time variables are also considered.

A long time ago Wey (1927) proposed to use a fractional power $\sqrt{m^2 - \nabla^2}$ of the modified Helmholtz operator $(m^2 - \nabla^2)$ in the relativistic problem. Weyl's idea of defining the operator corresponding to a symbol (Kohn and Nirenberg, 1965a; Weyl, 1927, pp. 27–28) was very similar to the concept of modern pseudodifferential operators (Kohn and Nirenberg, 1965b; see also Treves, 1980). Unfortunately Weyl did not develop a complete theory. The square-root operator approach to relativistic quantum theory was subsquently abandoned and new approaches were tried leading to the Klein–Gordon and the Dirac equations.

Recently, the square-root operator has been relevant in modern particle theory (Smith, 1993), in particular, in applications of the Bethe–Salpeter equation to bound states of quarks (Castorina *et al.,* 1984; Friar and Tomusiak, 1984; Nickisch *et al.,* 1984), in problems of binding in very strong fields (Hardekopf and Sucher, 1985; Papp, 1985), and in relativistic strings (bosonic) (Kaku, 1988; Fiziev, 1985). In this paper, we propose a very simple method allowing us to work with the square-root operator and to give its physical interpretation. Let us consider the Lagrangian form

[1] Institute of Physics and Technology, Mongolian Academy of Sciences, Ulaanbaatar 51, Mongolia.

$$L_\varphi^0 = \varphi^* (x) \sqrt{m^2 - \Box}\varphi(x) \tag{1}$$

where $\varphi(x)$ is some field operator whose nature for now is unknown. In accordance with the general rule ('t Hooft and Veltman, 1973), the propagators are minus the inverse of the operator found in the quadratic term (1). That is,

$$\tilde{D}(p) = -\frac{1}{\sqrt{m^2 - p^2 - i\varepsilon}} \tag{2}$$

Further, making use of the Dirac relation and the Feynman parameter method, one gets after changing integration variable

$$\tilde{D}(p) = -\frac{1}{(m - \hat{p})^{1/2}(m+\hat{p})^{1/2}} = \int_{-m}^{m} d\lambda\ \rho(\lambda)\tilde{S}(\lambda,\hat{p}) \tag{3}$$

where

$$\hat{S}(\lambda, \hat{p}) = \frac{1}{i}\frac{\lambda + \hat{p}}{\lambda^2 - p^2 - i\varepsilon} \tag{4}$$

is the spinor propagator with mass λ in momentum space and

$$\rho(\lambda) = \frac{1}{\pi}(m^2 - \lambda^2)^{-1/2} \tag{5}$$

In (3) we have used $\Gamma(1/2) = \sqrt{\pi}$. The function (5) possesses remarkable properties:

$$\int_{-m}^{m} d\lambda\rho(\lambda) = 1, \qquad \int_{-m}^{m} d\lambda\ \lambda\rho(\lambda) = 0, \qquad \int_{-m}^{m} d\lambda\ \lambda^2\rho(\lambda) = \frac{1}{2}m^2 \tag{6}$$

Equalities (3) and (6) mean that the propagator of the field $\varphi(x)$ defined by the Lagrangian (1) is exactly equal to the spinor propagator with random mass whose distribution is given by (5). In general, relation (3) solves the square-root differential operator problem in quantum field theory. Indeed, it is easily seen that solutions of the equations

$$\sqrt{m^2 - \Box}\varphi(x) = 0, \qquad \sqrt{m^2 - \Box}\varphi^*(x) = 0 \tag{7}$$

can be represented in the form

$$\varphi(x) = \int_{-m}^{\mathrm{m}} d\lambda\ \rho(\lambda)\psi(x,\lambda), \qquad \varphi^*(x) = \int_{-m}^{\mathrm{m}} d\lambda\ \rho(\lambda)\bar{\psi}(x,\lambda) \tag{8}$$

By definition the propagator of this field is

$$D(x-y) = \langle 0|T\{\varphi(x)\varphi^*(y)\}|0\rangle = \int_{-m}^{m}\int_{-m}^{m} d\lambda_1\, d\lambda_2\, \rho(\lambda_1)\rho(\lambda_2)\langle 0|T\{\psi(x,\lambda_1)\bar{\psi}(y,\lambda_2)\}|0\rangle \quad (9)$$

It is natural to assume that

$$\langle 0|T\{\psi(x,\lambda_1)\bar{\psi}(y,\lambda_2)\}|0\rangle = \delta(\lambda_1-\lambda_2)S(x-y,\lambda_1)/\rho(\lambda_1) \quad (10)$$

where $S(x, \lambda)$ is the spinor propagator of mass λ. Thus equation (9) yields relation (3), as it should. As seen below, the construction procedure for this extended theory is similar to the local one. Instead of (1) we use the Lagrangian density

$$L_\psi^0 = N\left\{\frac{i}{2}\left[\bar{\psi}(x,\lambda_1)\,\hat{\partial}\psi(x,\lambda_2) - \frac{\partial\bar{\psi}(x,\lambda_1)}{\partial x_\nu}\gamma^\nu\psi(x,\lambda_2)\right] - L_{1\psi}^o\right\} \quad (11)$$

or

$$L_\psi^0 = -N\{\bar{\psi}(x,\lambda_1)(-i\hat{\partial})\psi(x,\lambda_2) + L_{1\psi}^o\} \quad (12)$$

where we use the notation

$$L_{1\psi}^o = \bar{\Psi}(x,\lambda_1)U(\lambda_1,\lambda_2)\Psi(x,\lambda_2), \qquad \bar{\Psi}(x,\lambda_1) = (0, \bar{\psi}(x,\lambda_1))$$

$$N = \int_{-m}^{m}\int_{-m}^{m} d\lambda_1\, d\lambda_2\, \rho(\lambda_2)\rho(\lambda_2), \qquad \hat{\partial} = \gamma^\nu\frac{\partial}{\partial x^\nu}$$

$$\Psi(x,\lambda_2) = \begin{pmatrix} \psi(x,\lambda_2) \\ 0 \end{pmatrix}, \qquad U(\lambda_1,\lambda_2) = \begin{pmatrix} 0 & \lambda_1 \\ \lambda_2 & 0 \end{pmatrix}$$

Equations of motion can be obtained from the action

$$A = \int d^4x\, L_\psi^0(x)$$

by using independent variations over fields $\psi(y,\lambda)$ and $\bar{\psi}(y,\lambda)$ with the differences $\delta L_{1\psi}^o/\delta;\ \bar{\psi}(y,\lambda)$ and $\delta(L_{1\psi}^o)^T/\delta\psi(y,\lambda)$:

$$\int_{-m}^{m} d\lambda\, \rho(\lambda)(i\hat{\partial} - \lambda)\psi(x,\lambda) = 0,$$

$$\int_{-m}^{m} d\lambda\, \rho(\lambda)\left(i\frac{\partial\bar{\psi}(x,\lambda)}{\partial x^\nu}\gamma_\nu + \lambda\bar{\psi}(x,\lambda)\right) = 0 \quad (13)$$

Here we have used the obvious relations

$$\frac{\delta\overline{\psi}(x,\lambda_i)}{\delta\overline{\psi}(y,\lambda)} = \frac{\delta\psi(x,\lambda)}{\delta\psi(y,\lambda)} = \delta^{(4)}(x-y)\,\delta(\lambda_i-\lambda)$$

and the definition

$$(L^o_{1\psi})^T = \overline{\psi}(x,\lambda_1)U^T(\lambda_1,\lambda_2)\Psi(x,\lambda_2)$$

Let us introduce an electromagnetic interaction into this scheme. In order to ensure invariance of the Lagrangian (12) with respect to the local gauge transformation

$$\psi'(x,\lambda) = e^{ief(x)}\psi(x,\lambda) \qquad (14)$$
$$\overline{\psi}'(x,\lambda) = e^{-ief(x)}\overline{\psi}(x,\lambda)$$

it should be introduced into the gauge field $A_\mu(x)$ with the transformation rule

$$A'_\mu(x) = A_\mu(x) + \frac{\partial f}{\partial x^\mu} \qquad (15)$$

The standard procedure of changing $\partial_\mu\psi \to (\partial_\mu - ieA_\mu)\psi$ in (12) leads for our case to the interaction Lagrangian

$$L_{in}(x) = eN\{\overline{\psi}(x,\lambda_1)\hat{A}(x)\psi(x,\lambda_2\} \qquad (16)$$

where $\hat{A} = \gamma^\mu A_\mu(x)$. With (16) the S-matrix can be constructed by the usual rule:

$$S = \underset{\{\lambda i\}}{\text{Expec}}\; T\exp\left\{\int d^4x\, L_{in}(x)\right\} \qquad (17)$$

where the symbol T is the so-called T-product or T^*-operation and Expec means to take the expectation value over variables λ_i. Matrix elements of the S-matrix (17) are defined as products $\prod_{i\neq j} D(x_i - x_j)\gamma_i^\mu$ with γ_i^μ matrix coefficients, where $D(x)$ is given by (9). In our case, random variables λ_i entering into the definition of the spinor propagator with mass λ_i are not independent and have strong correlations between them. In other words, the functions $S(x,\lambda_i)$ are some stochastic processes over the variable λ_i. Expectation values of these processes are defined by the requirement of the gauge invariance of the theory and possess some properties such as white noise. For example, at least for connected diagrams in the momentum space, one assumes

$$Expec\{\tilde{D}(p)\} = \int_{-m}^{m} d\lambda\, \rho(\lambda)\tilde{S}(\hat{p},\lambda)$$

$$Expec\{\gamma^{v1}\tilde{D}(p_1)\gamma^{v2}\tilde{D}(p_2)\gamma^{v3}\}$$
$$= \frac{1}{2}\int_{-m}^{m}\int_{-m}^{m} d\lambda_1\, d\lambda_2\, \rho(\lambda_1)\rho(\lambda_2)$$

$$\times \{\gamma^{\nu 1}\tilde{S}(\hat{p_1}, \lambda_1)\gamma^{\nu 2}\tilde{S}(\hat{p_2}, \lambda_2)\gamma^{\nu 3}\} \times \left\{\frac{\delta(\lambda_1 - \lambda_2)}{\rho(\lambda_2)} + \frac{\delta(\lambda_1 - \lambda_2)}{\rho(\lambda_1)}\right\}$$

$$= \int_{-m}^{m} d\lambda\ \rho(\lambda)\{\gamma^{\nu 1}\tilde{S}(\hat{p_1}, \lambda)\gamma^{\nu 2}\tilde{S}(\hat{p_2}, \lambda)\gamma^{\nu 3}\} \tag{18}$$

$$Expec\{\gamma^{\nu_1}\tilde{D}(p_1)\gamma^{\nu_2}\tilde{D}(p_2)\gamma^{\nu_3}\tilde{D}(p_3)\gamma^{\nu_4}\}$$

$$= \frac{1}{3!}\int_{-m}^{m} d\lambda_1\, \rho(\lambda_1)\ldots\int_{-m}^{m} d\lambda_3\, \rho(\lambda_3)\{\gamma^{\nu_1}\tilde{S}(\hat{p_1}, \lambda_1)\lambda^{\nu_2}\tilde{S}(\hat{p_2}, \lambda_2)\gamma^{\nu_3}\tilde{S}(\hat{p_3}, \lambda_3)\gamma^{\nu_4}\}$$

$$\times \left\{\frac{\delta(\lambda_1 - \lambda_2)\delta(\lambda_2 - \lambda_3)}{\rho(\lambda_1)\rho(\lambda_3)} + \frac{\delta(\lambda_1 - \lambda_2)\delta(\lambda_1 - \lambda_3)}{\rho(\lambda_2)\rho(\lambda_3)} + \ldots\right\}$$

$$= \int_{-m}^{m} d\lambda\, \rho(\lambda)\{\gamma^{\nu_1}\tilde{S}(\hat{p_1}, \lambda)\gamma^{\nu_2}\tilde{S}(\hat{p_2}, \lambda)\gamma^{\nu_3}\tilde{S}(\hat{p_3}, \lambda)\gamma^{\nu_4}\}$$

etc. In the general case, one gets

$$Expec\{\gamma^{\nu_1}\tilde{D}(p_1)\gamma^{\nu_2}\ \ldots\ \gamma^{\nu_n}\tilde{D}(p_n)\gamma^{\eta_{n+1}}\}$$

$$= \int_{-m}^{m} d\lambda\ \rho(\lambda)\{\gamma^{\nu_1}\tilde{S}(\hat{p_1}\lambda)\gamma^{\nu_2}\ \ldots\ \gamma^{\nu_n}\tilde{S}(\hat{p_n}\lambda)\gamma^{\eta_{n+1}}\} \tag{19}$$

Definition (19) grants the gauge invariance of the theory. Indeed, in the language of perturbation theory (or Feynman diagrammatic techniques), the gauge invariance of the "square-root" QED means that every matrix element of the S-matrix defining the concrete electromagnetic processes has a definite structure, and algebraic relations exist between them. In particular, in the momentum representation, the so-called vacuum polarization diagram in the second order of perturbation theory has the form

$$\tilde{\Pi}_{\mu\nu}(k) = (k_\mu k_\nu - g_{\mu\nu}k^2)\Pi(k^2) \tag{20}$$

In addition, the relation

$$\frac{\partial\tilde{\Sigma}(p)}{\partial p_\mu} = -\ \tilde{\Gamma}_\mu(p, q)|_{q=0} \tag{21}$$

exists between the vertex function $\tilde{\Gamma}_\mu(p, q)$ and the self-energy of the "sqr-electron" $\tilde{\Sigma}(p)$. The relation (21) generalizes the Ward–Takahashi identity in QED. Here, in accordance with (18) and (19), we have

$$\tilde{\Sigma}(p) = \frac{-ie^2}{(2\pi)^4}\int_{-m}^{m} d\lambda\ \rho(\lambda)\int d^4k\triangle(k^2)\gamma^\mu\tilde{S}(\hat{p} - \hat{k}, \lambda)\gamma^\mu \tag{22}$$

and

$$\tilde{\Gamma}_\mu(p,q) = \frac{ie^2}{(2\pi)^4}\int d^4k\triangle((p-k)^2)\, Expec\,\{\gamma^\nu\tilde{D}(q+k)\gamma^\mu\tilde{D}(k)\gamma^\nu\}$$

$$= \frac{ie^2}{(2\pi)^4}\int_{-m}^{m} d\lambda\,\rho(\lambda)\int d^4k\triangle((p-k)^2)\gamma^\nu\tilde{S}(\hat{q}+\hat{k},\lambda)\gamma^\mu\tilde{S}(\hat{k},\lambda)\gamma^\nu \quad (23)$$

where $\tilde{S}(\hat{p},\lambda) = (\lambda - \hat{p})^{-1}$ and $\triangle(k^2) = (-k^2 - i\varepsilon)^{-1}$. For the proof of the relation (21) consider the identity

$$\frac{\partial\tilde{S}(\hat{p},\lambda)}{\partial p_\mu} = \tilde{S}(\hat{p},\lambda)\gamma^\mu\tilde{S}(\hat{p},\lambda) \quad (24)$$

Here the vertex γ^μ is given by

$$\gamma^\mu = -\frac{\partial}{\partial p_\mu}\tilde{D}^{-1}(p), \qquad \tilde{D}^{-1}(p) = \int_{-m}^{m} d\lambda\,\rho(\lambda)(\lambda - \hat{p})$$

This definition follows from the identity

$$Expec\{\tilde{D}(p)\tilde{D}^{-1}(p)\} = \int_{-m}^{m}\int_{-m}^{m} d\lambda_1\, d\lambda_2\,\rho(\lambda_1)\rho(\lambda_2)\tilde{S}(\hat{p}_1,\lambda_1)\tilde{S}^{-1}(\hat{p}_2,\lambda_2) \times \frac{\delta(\lambda_1-\lambda_2)}{\rho(\lambda_1)}$$

$$= \int_{-m}^{m} d\lambda\,\rho(\lambda)\tilde{S}(\hat{p},\lambda)\tilde{S}^{-1}(\hat{p},\lambda)$$

$$= \int_{-m}^{m} d\lambda\,\rho(\lambda) = 1$$

Further, it is easy to verify the identity (21) by differentiating (22) over p_μ and making use of the equality (24), and choosing other momentum variables in (23) and assuming $q = 0$, $p' = p + q = p$. Relations of the type $q_\mu\tilde{\Gamma}_\mu(p, q)|_{p'^2=p^2=\lambda^2} = 0$ follow from the definition

$$q_\mu\, Expec\{\tilde{D}(p_1)\gamma^\mu\tilde{D}(p_2)\}$$

$$= q_\mu\int_{-m}^{m}\int_{-m}^{m} d\lambda_1\, d\lambda_2\,\rho(\lambda_1)\rho(\lambda_2) \times \tilde{S}(\hat{p}_1,\lambda_1)\gamma^\mu\tilde{S}(\hat{p}_2,\lambda_2)\frac{\delta(\lambda_1-\lambda_2)}{\rho(\lambda_1)}$$

$$= \tilde{D}(p_1) - \tilde{D}(p_2)$$

$$= \int_{-m}^{m} d\lambda\ \rho(\lambda)[\tilde{S}(\hat{p_1}, \lambda) - \tilde{S}(\hat{p_2}, \lambda)]$$

if $q = p_1 - p_2$.

Now let us demonstrate the gauge invariance of the photon self-energy diagram in the "square-root" QED; its matrix element is given by

$$\tilde{\Pi}_{\mu\nu}(K) = e^2\ Expec\left\{\int d^n p\ \mathrm{Tr}[\gamma^\mu \tilde{D}(p+k)\gamma^\nu \tilde{D}(p)]\right\}$$
$$= e^2 \int_{-m}^{m} d\lambda\ \rho(\lambda) \int d^n p\ \mathrm{Tr}[\gamma^\mu \tilde{S}(\hat{p} + \hat{k}, \lambda)\gamma^\nu \tilde{S}(\hat{p}, \lambda)] \quad (25)$$

Here we have used the n-dimensional gauge-invariant regularization procedure due to 't Hooft and Veltman (1972) and definition (18). After some calculations we obtain the same form as (20),

$$\Pi_{\mu\nu}(k) = \frac{8i\pi^{n/2}}{\Gamma(2)}\Gamma\left(2 - \frac{1}{2}n\right)(k_\mu k_\nu - k^2 g_{\mu\nu})\int_{-m}^{m} d\lambda\ \rho(\lambda)$$
$$\times \int_0^1 dx \cdot x(1-x)[\lambda^2 - k^2 x(1-x)]^{n/2-2} \quad (26)$$

which is manifestly gauge invariant.

Now we attempt to shed light on the physical origin of the appearance of an extended object (8) over random mass with propagator (3). There is a common rule that in the static limit the propagator of force-transmitting quanta is related to the potential of this field. We know that the propagators of the photon and the scalar particle with mass m are defined by inverse Fourier transforms of the corresponding Coulomb and Yukawa potentials. It turns out that the potential origin of the square root operator field arises from dipolelike extended objects. For example, the usual electric dipole potential

$$U_d(r) = \frac{ed\ cos\ \theta}{4\pi\varepsilon_0 r^2} \quad (27)$$

is associated with the Fourier transform of minus the propagator $-\tilde{D}_d^0 = 1/\sqrt{\mathbf{p}^2}$, the four-dimensional version of which is

$$\tilde{D}_d^0(p) = -\frac{1}{\sqrt{-p^2}} = i\frac{\hat{p}}{p^2} \quad (28)$$

The latter means that the dipole field may emit or absorb a "nonlocal photon" or more precisely a neutrino-like particle. For definiteness, we call it a photino, like the fermion partner of the photon in the supersymmetric theory (Bailin and Love, 1994). By analogy with the introduction of the Yukawa potential, the short-distance modification of the dipole potential

$$U_d^m(r) = \frac{eLm}{2\pi^2 r} K_1(mr) \qquad (L = \pi d/2) \tag{29}$$

results minus the propagator $-\tilde{D}_d^m(p) = \tilde{D}(p)$ in (2). Here $K_1(x)$ is the modified Bessel function. The appearance of extended objects over space-time variables

$$\varphi_l(x) = \int d^4y\, K_l^m(\square)\delta^{(4)}(x-y)\varphi(y) = \int d^4y K_l^m(x-y)\varphi(y) \tag{30}$$

in (8) is caused by a more general potential

$$U_{dl}^m(r) = \frac{eL}{2\pi^2}\frac{m}{\sqrt{r^2+l^2}} K_1(m\sqrt{r^2+l^2}) \tag{31}$$

where l is a parameter of the theory; we call it the fundamental length. The propagator of the field (30) in momentum space is given by

$$\tilde{D}_m^l(p) = -\frac{1}{\sqrt{m^2 = p^2 - i\,\varepsilon}}\, exp(-l\sqrt{m^2-p^2}) \tag{32}$$

in equation (30)

$$K_l^m(x) = \frac{l}{8\pi^2}\sqrt{\frac{2}{\pi}}\, m^{5/2}\, K_{5/2}\left(m\sqrt{\frac{1}{4}l^2 - x^2}\right) \times \left(\frac{1}{4}l^2 - x_2\right)^{-5/4}, \qquad x^2 = x_0^2 - \mathbf{x}^2 \tag{33}$$

is a generalized function and its Fourier transform in the Euclidean space reads

$$\tilde{K}_l^m(p_E) = \int d^4x_E\, e^{-ip_Ex_E}\, K_l^m(x_E) = \exp\left[-\frac{l}{2}\sqrt{m^2+p_E^2}\right] \tag{34}$$

For the propagator (32) the Mellin representation

$$\tilde{D}_m^l(p) = \frac{l}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi\, \frac{\upsilon(\xi)}{\sin\pi\xi}\,[l^2(m^2-p^2)]_\xi \tag{35}$$

$$v(\xi) = \frac{1}{\cos\pi\xi}\frac{1}{\Gamma(2+2\xi)} \qquad (1 \le \beta < 1/2) \tag{36}$$

is valid.

The square-root operator field theory with the propagator (35) will be studied elsewhere. The requirement of invariance of the Lagrangian (12) with respect to nonlocal gauge transformations [instead of (14) and (15)]

$$
\begin{aligned}
\psi'(x, \lambda) &= \exp\left[ie \int d^4y \, K_l^0(x - y) f(y) \right] \psi(x, \lambda) \\
\overline{\psi'}(x, \lambda) &= \exp\left[-ie \int d^4y \, K_l^0(x - y) f(y) \right] \overline{\psi}(x, \lambda) \\
A'_\mu(x) &= A_\mu(x) + \frac{\partial f}{\partial x^\mu}
\end{aligned}
\tag{37}
$$

gives rise to the introduction of the nonlocal photon field

$$A^l_\mu(x) = \int d^4y \, K_l^0(x - y) A_\mu(y) \tag{38}$$

in the square-root operator formalism. Thus the interaction Lagrangian (16) acquires the nonlocal character

$$L^l_{in}(x) = eN\{\overline{\psi}(x, \lambda_1)\hat{A}^l(x)\psi(x, \lambda_2\} \tag{39}$$

Construction of a such theory is similar to nonlocal quantum electrodynamics for a pointlike spinor in the Efimov theory (Efimov, 1977; Namsrai, 1986). In expressions (37) and (38) the nonlocal generalized function $K_l^0(x)$ is given by

$$(K_l^0(x) = \frac{3l}{8\pi^2} \left(\frac{1}{4} l^2 - x^2 \right)^{-5/2} \tag{40}$$

the Fourier transform of which is equal to

$$\tilde{K}_l^0(p) = \exp\left(-\frac{l}{2} \sqrt{p^2} \right) \tag{41}$$

The physical meaning of (40) is the distribution of the ring (closed string) charge e of the spinor field $\psi(x, \lambda)$ in $\mathbf{x}$ space. Thus, the propagator of the nonlocal photon takes the form

$$D^l_{\mu\nu}(x - y) = \frac{-g_{\mu\nu}}{(2\pi)^4 i} \int d^4k \, \frac{[\tilde{K}_l^0(p)]^2 \, e^{-ik(x-y)}}{-k^2 - i\varepsilon} \tag{42}$$

The nonlocal theory with the interaction Lagrangian (39) and propagators (3) and (42) is gauge invariant by construction. Investigation of the matrix elements for the S-matrix in this theory will be presented elsewhere.

Finally, we mention that the square-root operator field equations (7) with the solutions (8) and the propagators (3) may be able to shed light on the physical origin of mass and indicate that in the microworld the mass value of particles is not quantized, but takes stochastic distributional character through the mechanism of their generation. This is a problem for future study.

REFERENCES

Bailin, D., and Love, A. (1994). *Supersymmetric Gauge Field Theory and String Theory*, Institute of Physics, Bristol, England.

Castorina, P., Cea, P., Nardulli, G., and Paiano, G. (1984). *Physical Review D*, **29**, 2660.

Efimov, G. V. (1977). *Nonlocal Interactions of Quantized Fields*, Nauka, Moscow.

Fiziev, P. P. (1985). *Theoretical and Mathematical Physics*, **62**, 123.

Friar, J. L., and Tomusiak, E. L. (1984). *Physical Review C*, **29**, 1537.

Hardekopf, G., and Sucher, J. (1985). *Physical Review A*, **31**, 2020.

Kaku, M. (1988). *Introduction to Superstrings*, Springer-Verlag, Berlin, pp. 48–58.

Kohn, J. J., and Nirenberg, L. (1965a). *Communications in Pure and Applied Mathematics*, **18**, 293–295.

Kohn, J. J., and Nirenberg, L. (1965b). *Communications in Pure and Applied Mathematics*, **18**, 304.

Namsrai, Kh. (1986). *Nonlocal Quantum Field Theory and Stochastic Quantization*, Reidel, Dordrecht.

Nickisch, L. J., Durand, L., and Durand, B. (1984). *Physical Review D*, **30**, 660.

Papp, E. (1985). Quasiclassical upper bounds on the nuclear charge, Munich-preprint.

Smith, J. R. (1993). Second quantization of the square-root Klein–Gordon operator, microscopic causality, propagators, and interaction, Preprint UCD/II RPA 93-13, University of California, Davis.

’t Hooft, G., and Veltman, M. (1972). *Nuclear Physics B*, **44**, 189–213.

’t Hooft, G., and Veltman, M. (1973). Diagrammar, CERN Preprint 73-9, Geneva.

Treves, F. (1980). *Introduction to Pseudodifferential and Fourier Integral Operators*, Plenum Press, New York, Vol. 1, p. 20.

Weyl, H. (1972). *Zeitschrift für Physik*, **46**, 38–44.